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FLOW SYMMETRY DISTURBANCE DUE TO THERMAL INSTABILITY

V. V. Grachev and É. N. Rumanov

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The instability of the laminar mode associated with the origin of turbulence is usually inessential for flows of a strongly viscous fluid since the Reynolds numbers are small. In the forefront for such flows is the thermal instability detected and investigated in [1, 2] and elsewhere. It was shown in [2] that the thermal instability holds for the pressure drop Δp determined and results in jumps in the flow rate for definite critical values of Δp (hydrodynamic inflammation and extinction).

Meanwhile, a thermal instability also occurs in electrical systems even for a fixed current (the analog of the mass flow in hydrodynamics), where the development of the instability results in inhomogeneous [3] or nonsymmetric [4] modes.

Disturbance of the flow symmetry through a pair of tubes (connected in parallel) is considered in this paper for a fixed total mass flow rate. It is shown that in contrast to an analogous electrical system [4], the disturbed symmetry is restored in the case under consideration as the flow rate increases. Restoration of the symmetry is due to the convective nature of the instability.

1. The flow of a strongly viscous incompressible fluid through two cylindrical tubes connected in parallel and with a given total flow rate is considered. As the fluid moves, heat is liberated because of dissipation and is eliminated in the tube walls.

The following equations hold for tubes connected in parallel

$$\frac{\pi r^4}{8}\Delta p = Q_1\int_0^l \mu(T_1)\,dz = Q_2\int_0^l \mu(T_2)\,dz, \quad Q = Q_1 + Q_2,$$

where Δp is the pressure drop between the entrance into, and exit from the tubes, r is the tube radius, l is the tube length, z is the coordinate along the tube axes, μ is the dynamic viscosity, T_1 and T_2 are the fluid temperatures, Q_1 and Q_2 are the mass flow rates, the subscripts 1 and 2 refer, respectively, to the first and second tubes, and Q is the total mass flow rate which is a given quantity. As regards the equation in the temperature, then under the conditions

$$\mathrm{Pr} = \mu/(\rho\chi) \gg 1,\ \mathrm{Pe} = Ql/(\pi r^2\chi) \gg 1,\ \mathrm{Bi} = \alpha r/(c\rho\chi) \ll 1,$$

where χ is the thermal diffusivity coefficient, ρ is the density, c is the specific heat of the fluid, α is the coefficient of heat transfer, Pr is the Prandtl criterion, Pe is the Peclet criterion, Bi is the Biot criterion, they take a form analogous to (1.11) in [2] for both tubes. In dimensionless variables

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$$\Theta = \frac{U}{RT_0^2}(T - T_0), \quad \xi = \frac{z}{l}, \quad \tau = \frac{t}{t_0}, \quad t_0 = \frac{8l^2\mu(T_0)}{c\rho r^2}\frac{U}{RT_0^2}$$

the system under consideration becomes

$$\frac{\partial\Theta_i}{\partial\tau} + \omega_i\frac{\partial\Theta_i}{\partial\xi} = \omega_i^2 e^{-\Theta_i} - B\Theta_i, \quad i = 1, 2; \tag{1.1}$$

$$\omega_1\int_0^1 e^{-\Theta_1}d\xi = \omega_2\int_0^1 e^{-\Theta_2}d\xi; \tag{1.2}$$

$$\omega = \omega_1 + \omega_2, \tag{1.3}$$

where

$$\omega_i = \frac{8l\mu(T_0)}{c\rho\pi r^4}\frac{U}{RT_0^2}Q_i; \quad B = \frac{16l^2\mu(T_0)}{(c\rho)^2 r^3}\frac{U}{RT_0^2}\alpha;$$

$$\mu = \mu_0 \exp(U/RT);$$

and T_0 is the temperature of the surrounding medium. The expansion of the exponential exp (U/RT) in [5] is used for the nondimensionalization.

2. If the condition

$$\frac{\tau_1}{\tau_2} \sim \frac{\omega}{B} = \frac{\mathrm{Pe}}{\mathrm{Bi}}\frac{r^2}{2l^2} \ll 1, \quad \tau_1 \sim \frac{rc\rho}{\alpha}, \quad \tau_2 \sim \frac{\pi r^2 l}{Q} \tag{2.1}$$

is satisfied, where τ_1 is the characteristic heat-transfer time, τ_2 is the fluid residence time in the tube, then the entrance section of the tube whose length is $\xi_0 \sim \tau_1\omega \ll 1$ can be neglected and the temperature of the fluid along the tube can be considered constant, and $\partial\Theta_i/\partial\xi = 0$. Then the system (1.1)-(1.3) results in the form

$$\frac{d\Theta_1}{d\tau^0} = \varkappa e^{\Theta_1}\left(e^{\Theta_1} + e^{\Theta_2}\right)^{-2} - \Theta_1; \tag{2.2}$$

$$\frac{d\Theta_2}{d\tau^0} = \varkappa e^{\Theta_2}\left(e^{\Theta_1} + e^{\Theta_2}\right)^{-2} - \Theta_2. \tag{2.3}$$

Here the flow rates ω_1 and ω_2 are expressed in terms of the total flow rate by using (1.2) and (1.3) and the new dimensionless time $\tau^0 = B\tau$ is introduced while the parameter is

$$\varkappa = \frac{\omega^2}{B} = Q^2\frac{U}{RT_0^2}\frac{4\mu(T_0)}{\pi^2 r^5\alpha}.$$

An investigation of the phase trajectories of the system (2.2) and (2.3) showed [4] that for any values of the parameter $\varkappa$ asymmetric stationary mode $\Theta_1 = \Theta_2$ exists that is stable (stable "node") for $\varkappa < 4e$ and unstable ("saddle") for $\varkappa > 4e$. Moreover, for $\varkappa > 4e$ two more nonsymmetric $\Theta_1 \neq \Theta_2$ stable stationary models (stable "nodes") exist. For $\varkappa = 4e$ the nonsymmetric modes merge with the symmetric mode.

Therefore, there is a critical value of the parameters $\varkappa_* = 4e$ and a correspondingly critical value of the total mass flow rate Q_*. If $Q < Q_*$, then the fluid temperatures in both tubes are equal and the symmetric mode $Q_1 = Q_2 = Q/2$ is realized. If $Q > Q_*$, then the stationary mode with different flow rates and temperatures is unstable. The spontaneous disturbance of the flow symmetry occurs, the system goes over into a new stationary mode when the fluid temperatures in the tubes are distinct, and a major part of the fluid flows through the tube in which the temperature is higher. The disturbance of the symmetry occurs for $Q = Q_*$ in the soft mode.

3. The inequality (2.1) is not satisfied for high flow rates. In this case heat entrainment by the fluid stream becomes essential, and it is already impossible to neglect the convective term in (1.1).

In the opposite limit case $\tau_1/\tau_2 \gg 1$, the heat transfer in the tube walls can be neglected, $B\Theta_i = 0$. Then the system (1.1)-(1.3) with the boundary condition

$$\xi = 0 : \Theta_1 = \Theta_2 = 0 \tag{3.1}$$

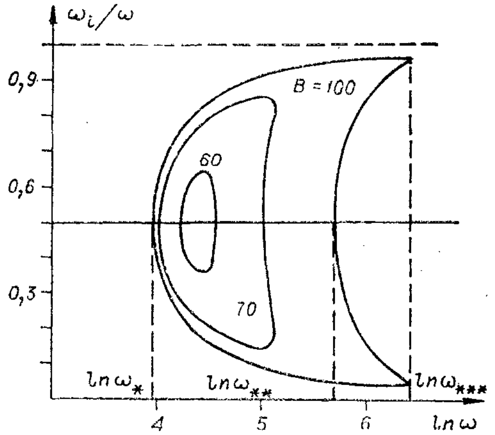

Fig. 1

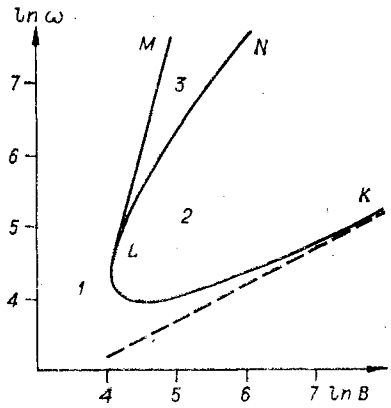

Fig. 2

has the unique stationary solution $\Theta_1 = \Theta_2 = \ln(1 + \omega\xi/2)$, where this solution is stable to small perturbations.

Therefore, only one (symmetric) stationary mode is possible. Hence, a second critical value of the total rate Q_{**} should exist for which the symmetric mode again becomes stable.

Restoration of the flow symmetry is due to the convective nature of the thermal instability in this system. In the domain of high flow rates, perturbations that do not succeed in being developed are carried outside the system limits.

4. To find the conditions for restoration of flow symmetry a numerical solution of the problem was carried out on a computer.

Stationary solutions of the system (1.1)-(1.3) with the boundary conditions (3.1) were found. The stability of the stationary solutions was investigated by using a linearized system of equations for the small deviations $v_i(\xi)e^{\lambda\tau}$, $w_i e^{\lambda\tau}$ from the stationary solutions $\Theta_i(\xi)$, ω_i (i = 1, 2):

$$\omega_i \frac{dv_i}{d\xi} = -\left(\lambda + B + \omega_i e^{-\Theta_i}\right) v_i + \left(\omega_i e^{-\Theta_i} + \frac{B}{\omega_i}\Theta_i\right) w_i,$$

$$\int_0^1 (w_1 - \omega_1 v_1)\, e^{-\Theta_1} d\xi = \int_0^1 (w_2 - \omega_2 v_2)\, e^{-\Theta_2} d\xi,$$

$$w_1 + w_2 = 0,\ \xi = 0{:}\ v_1 = v_2 = 0.$$

The results of the numerical computation are represented in Figs. 1 and 2. The dependences of the ratio between the mass flow rate through one of the tubes and the total given flow rate on the magnitude of the total flow rate are shown in Fig. 1 for different values of the heat-transfer coefficient B.

For B = 100 and $\omega < \omega_* = 26.47$ only a symmetric mode with equal flow rates $\omega_1 = \omega_2 = \omega/2$ exists.

For $\omega = \omega_*$ the spontaneous disturbance of flow symmetry occurs in the soft mode. Furthermore, as the total flow rate increases the flow rate in one of the tubes rises while it drops in the other. For $\omega = \omega_{**} = 149.8$ the symmetric flow mode becomes stable, and two other unstable nonsymmetric modes occur. As the total flow rate increases further, the stable and unstable nonsymmetric modes approach each other, and merge for $\omega = \omega_{***} = 617.6$ and vanish. Hence, if the total flow rate diminishes, then disturbance of the symmetry will occur in the hard mode for $\omega = \omega_{**}$.

As the heat-transfer coefficient B diminishes, the differences $(\omega_{**} - \omega_*)$ and $(\omega_{***} - \omega_{**})$ diminish. For $B < B_* = 58.82$ and any values of the total flow rate, only a symmetric flow mode is possible, the critical phenomena are degenerate.

Partition of the plane of the parameters B, ω into domains with distinct stationary modes is represented in Fig. 2. Only a symmetric stable flow mode exists in domain 1, an unstable symmetric mode and two stable nonsymmetric modes in domain 2, and a stable symmetric mode and four nonsymmetric modes, two of which are stable and two unstable, are in domain 3. The curve KLN corresponds to the stability boundary for the symmetric mode, and the curve LM to merger of the nonsymmetric modes and their disappearance. The dashed line corresponds to an approximate condition for disturbance of the symmetry ($\varkappa_* = 4e$) according to (2.2) and (2.3).

It is interesting that the instability domain of the symmetric mode NLK corresponds to the bistability domain obtained in [2].

Taking account of the results obtained for the flow considered, it can be assumed that a thermal instability can result in flow partition into a jet in the filtration of a strongly viscous fluid in a mode with determination of the total flow rate. Thermal wave propagation across the filtering stream is possible in case of head determination.

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MIXED LAMINAR CONVECTION AROUND A VERTICAL CYLINDER WITH A CONSTANT SURFACE TEMPERATURE

Yu. P. Semenov

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The heat exchange accompanying mixed convection around vertical cylinders plays an important role in many technological processes and in the operation of power plants. However, this problem has not been studied to a sufficient extent. Most papers concerned with mixed laminar convection are devoted to processes on vertical flat surfaces [1, 2] and horizontal cylinders [3]. The heat exchange due to mixed convection from vertical cylinders has been investigated in individual papers only. The problem of concurrent mixed convection from a vertical cylinder with a constant surface temperature is solved in [4] by means of the method of local non-self-similarity, which is also used in [5] for solving this problem for a constant thermal flux q_w. Mixed convection from a vertical cylinder for q_w = const was investigated experimentally and numerically in [6, 7]. Thus, mixed laminar convection from vertical cylinders for t_w = const has been investigated only in [4]. However, the mixed convection parameter varied there in the limited range $0 \leqslant Gr/Re^2 \leqslant 2$; generalized theoretical relationships were not derived, and only concurrent convection was contemplated. This made it necessary to undertake the investigation described here.

Mixed convection from vertical cylinders constitutes a non-self-similar problem. The method of local non-self-similarity used in [4, 5] is approximate. In order to obtain the solution for the entire region of mixed convection, it is necessary to solve the boundary layer equations written in terms of self-similar variables of forced motion in a region close to the forced motion and the boundary layer equations written in terms of self-similar variables of natural convection in a region close to the natural convection. In this case, it is more advisable to obtain directly the numerical solution of the boundary layer equations.

The method described in [8, 9] is used for solving numerically the problem of mixed convection from a vertical cylinder. This method was used in [7] for investigating mixed convection from a vertical cylinder for q_w = const and in [10] for investigating natural convection.

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